**What are rational numbers?**

Rational numbers are numbers that can be written as the quotient of two integers. Since the bar in a fraction represents division, every fraction whose numerator and denominator is an integer is a rational number.

Any number that could be written as a fraction whose numerator and denominator is an integer is also a rational number.

Think

Every integer, whole number, and natural number is a rational number.

You can write every integer, whole number, and natural number as a fraction. So they are all rational numbers. The square root of a perfect square is also a rational number.

\[
\sqrt{1} = 1, \quad \sqrt{4} = 2, \quad \sqrt{9} = 3, \quad \sqrt{16} = 4
\]

\[
3 = \frac{3}{1}, \quad -5 = \frac{-5}{1}, \quad 0 = \frac{0}{1}, \quad \sqrt{25} = 5 \text{ or } \frac{5}{1}
\]

Think

Every terminating decimal is a rational number.

You can write every terminating decimal as a fraction. So terminating decimals are all rational numbers.

You can use what you know about place value to find the fraction that is equivalent to any terminating decimal.

\[
0.4 = \frac{4}{10} = \frac{2}{5} \\
0.75 = \frac{75}{100} = \frac{3}{4} \\
0.386 = \frac{386}{1000} = \frac{193}{500} \\
\sqrt{0.16} = 0.4 = \frac{4}{10} = \frac{2}{5}
\]
Part 1: Introduction

Think  Every repeating decimal is a rational number.

You can write every repeating decimal as a fraction. 
So repeating decimals are all rational numbers.

As an example, look at the repeating decimal 0.3.

Let  \[ x = 0.3 \]

\[ 10 \cdot x = 10 \cdot 0.3 \]

\[ 10x = 3.3 \]

The repeating pattern goes to the tenths place. Multiply both sides by 10.

\[ 10x - x = 3.3 - 0.3 \]

\[ 9x = 3 \]

\[ x = \frac{3}{9} \] or \[ \frac{1}{3} \]

\[ 0.3 = \frac{1}{3} \]

You can write and solve an equation to find a fraction equivalent to a repeating decimal.

Here’s another example of how you can write a repeating decimal as a fraction.

\[ x = 0.512 \]

\[ 1,000x = 512.512 \]

The repeating pattern goes to the thousandths place. Multiply by 1,000.

\[ 1,000x - x = 512.512 - 0.512 \]

Subtract \( x \) from the left side and the repeating decimal from the right side.

\[ 999x = 512 \]

\[ x = \frac{512}{999} \]

Reflect

1 What fraction is equivalent to 5.1? Is 5.1 a rational number? Explain.
Explore It

What numbers are not rational? Let’s look at a number like $\sqrt{2}$, the square root of a number that is not a perfect square.

2 Look at the number line below. The number $\sqrt{2}$ is between $\sqrt{1}$ and $\sqrt{4}$. Since $\sqrt{1} = 1$ and $\sqrt{4} = 2$, that means that $\sqrt{2}$ must be between what two integers?

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3 Draw a point on the number line where you would locate $\sqrt{2}$. Where did you draw the point?

4 Calculate: $1.3^2 = \underline{\phantom{00}}$ $1.4^2 = \underline{\phantom{00}}$ $1.5^2 = \underline{\phantom{00}}$

5 Based on your calculations, draw a point on the number line below where you would locate $\sqrt{2}$ now. Where did you draw the point?

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6 Calculate: $1.41^2 = \underline{\phantom{00}}$ $1.42^2 = \underline{\phantom{00}}$

7 Based on these calculations, $\sqrt{2}$ is between which two decimals? ______________

8 You can continue to estimate, getting closer and closer to the value of $\sqrt{2}$. For example, $1.414^2 = 1.999396$ and $1.415^2 = 2.002225$, but you will never find an exact number that multiplied by itself equals 2. The decimal will also never have a repeating pattern.

$\sqrt{2}$ cannot be expressed as a terminating or repeating decimal, so it cannot be written as a fraction. Numbers like $\sqrt{2}$ and $\sqrt{5}$ are not rational. You can only estimate their values. They are called irrational numbers. Here, irrational means “cannot be set as a ratio.” The set of rational and irrational numbers together make up the set of real numbers.

Now try this problem.

9 The value $\pi$ is a decimal that does not repeat and does not terminate. Is it a rational or irrational number? Explain.
Part 2: Guided Instruction

Lesson 3

Talk About It

You can estimate the value of an irrational number like $\sqrt{5}$ and locate that value on a number line.

10 $\sqrt{5}$ is between which two integers? Explain your reasoning.

11 Mark a point at an approximate location for $\sqrt{5}$ on the number line below. $\sqrt{5}$ is between which two decimals to the tenths place? _____________

12 Calculate: $2.2^2 = _______\enspace 2.23^2 = _______\enspace 2.24^3 = _______

   Based on your results, $\sqrt{5}$ is between which two decimals to the hundredths place? _____________

13 Draw a number line from 2.2 to 2.3. Label tick marks at tenths to show 2.21, 2.22, 2.23, and so on. Mark a point at the approximate location of $\sqrt{5}$ to the hundredths place.

Try It Another Way

Explore using a calculator to estimate irrational numbers.

14 Enter $\sqrt{5}$ on a calculator and press Enter. What is the result on your screen? _____________

15 If this number is equal to $\sqrt{5}$, then the number squared should equal ______.

16 Clear your calculator. Then enter your result from problem 14. Square the number. What is the result on your screen? _____________

17 Explain this result.
   ________________________________________________________________________________
   ________________________________________________________________________________
Connect It

Talk through these problems as a class, then write your answers below.

18 **Illustrate:** Show that 0.74 is equivalent to a fraction. Is 0.74 a rational or irrational number? Explain.

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19 **Analyze:** A circle has a circumference of $3\pi$ inches. Is it possible to state the exact length of the circumference as a decimal? Explain.

________________________________________________________________________

________________________________________________________________________

20 **Create:** Draw a Venn diagram showing the relationships among the following sets of numbers: integers, irrational numbers, natural numbers, rational numbers, real numbers, and whole numbers.
Part 4: Common Core Performance Task

Lesson 3

Put It Together

Use what you have learned to complete this task.

21 Consider these numbers:

\[
\sqrt{50}, \quad 3.4\overline{56}, \quad 0, \quad \sqrt{\frac{4}{9}}, \quad 0.38, \quad \sqrt{81}, \quad 2\pi, \quad \sqrt{1.69}, \quad \sqrt{\frac{7}{9}}
\]

A Write each of the numbers in the list above in the correct box.

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>Irrational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B Circle one of the numbers you said was rational. Explain how you decided that the number was rational.

________________________________________________________________________

________________________________________________________________________

C Now circle one of the numbers you said was irrational. Explain how you decided that the number was irrational.

________________________________________________________________________

________________________________________________________________________

D Draw a number line and locate the two numbers you circled on the line. Write a comparison statement using <, =, or > to compare the numbers.

________________________________________________________________________