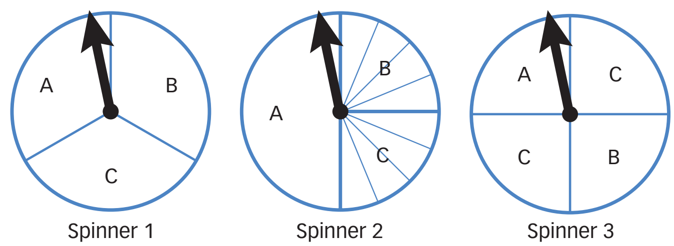
Lesson 32 Part 1: Introduction

Probability Models

CCLS
7.SP.C.7a
7.SP.C.7b

In Lesson 31, you learned about probability based on an experiment. Consider this problem.

A game at a school carnival has three different spinners. The spinners are shown. Which spinner is most likely to land on section A?



Explore It

Use the math you already know to solve the problem.

- How can you set up a ratio to represent section A on Spinner 1? What fraction of Spinner 1 is represented by section A?

  ____

- Is the pointer on Spinner 1 more likely to land on section A than on B or C? Explain.

  ____

- What fraction of Spinner 2 is represented by section A? __________

- Is the pointer on Spinner 2 more likely to land on section A than on B or C? Explain.

  ____

- Which section is Spinner 3 most likely to land on? Explain. ______________

  ____

- Explain how to use ratios to decide which spinner is most likely to land on section A.

  ____

  ____

Find Out More

When you find the probability of an event based on the results from an experiment, you are finding the **experimental probability** of that event. The probabilities you found in Lesson 31 are experimental probabilities.

$$\text{experimental probability of an event} = \frac{\text{number of times event occurs}}{\text{number of trials}}$$

The **theoretical probability** of an event is what you would expect to happen in an experiment. You can use theoretical probability to find the likelihood of an event.

The **sample space** is the set of possible outcomes for an experiment. For all of the spinners, the sample space is the set of letters A, B, and C. When the probability model is **uniform** and each outcome is equally likely, you can use the following ratio:

$$\text{theoretical probability of an event} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

For Spinners 1 and 3, each section is equally likely, so those probability models are uniform.

The favorable outcome is the letter section of interest. The number of outcomes is the number of sections on each spinner.

For example: $P(\text{C on Spinner 1}) = \frac{\text{sections lettered C}}{\text{number of sections}} = \frac{1}{3}$

$P(\text{C on Spinner 3}) = \frac{\text{sections lettered C}}{\text{number of sections}} = \frac{2}{4} = \frac{1}{2}$

For Spinner 2, each section is not equally likely. The probability model is **non-uniform**. Section A is larger than sections B and C. For this spinner, compare the favorable section to the whole to find the probability.

On Spinner 2, section A represents $\frac{1}{2}$ of the spinner, so $P(\text{A on Spinner 2}) = \frac{1}{2}$.

Which spinner is most likely to land on the letter A? Find each theoretical probability.

$P(\text{A on Spinner 1}) = \frac{1}{3}$ $P(\text{A on Spinner 2}) = \frac{1}{2}$ $P(\text{A on Spinner 3}) = \frac{1}{4}$

One-half is greater than $\frac{1}{3}$ or $\frac{1}{4}$. Spinner 2 is most likely to land on the section with letter A.

Reflect

1 Which spinner is more likely than the others to land on a section with letter B? Explain.

Read the problem below. Then explore different ways to understand it.

A family has three girls. The family tossed a coin to represent the likelihood of a girl if they have another child. They let heads represent a girl and tails represent a boy. The table shows the results of their experiment.

Coin Toss	Tally	Total
Heads (Girl)	卌 |||	8
Tails (Boy)	卌 卌 ||	12

If the family has another child, predict whether the child will be a girl.

Picture It

Describe the sample space.

The sample space consists of two possible equally likely outcomes: girl or boy.

The favorable outcome is a girl.

Model It

Compare the experimental probability to the theoretical probability.

Experimental probability of a girl:

Use the results of the experiment.

$$P(\text{Girl}) = \frac{\text{number of heads}}{\text{number of tosses}} = \frac{8}{20}$$

Theoretical probability of having a girl:

number of equally likely possible outcomes: 2 (girl or boy)

favorable outcomes: 1 (girl)

$$P(\text{Girl}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{2}$$

The experimental probability of a girl is less than the theoretical probability of a girl.

Connect It

Now you will solve the problem from the previous page using probability.

2 Based on the results of the family experiment, predict whether the next child will be a girl. Explain.

____

____

3 Repeat the experiment by tossing the coin 20 times. Record your results in the chart.

Coin Toss	Tally	Total
Heads (Girl)		
Tails (Boy)		

4 Based on your results, what is the experimental probability of a girl? ______________

5 Compare the experimental probability from problem 3 to the theoretical probability of having a girl.

____

____

6 Will the experimental and theoretical probability of an event always differ? Why or why not?

____

____

Try It

Use what you learned to solve this problem.

7 What is the theoretical probability that the child will be a boy? ______________

8 Does the fact that the family has 3 girls change the probability that the next child is a girl? Explain.

____

____

Read the problem below. Then explore different ways to understand it.

Serena is playing a game with a number cube. To win the game, she must roll the number 4 on her next roll. The table shows the number of times she has rolled each number so far.

Number	Number of Rolls
1	3
2	6
3	2
4	5
5	4
6	4

What is the probability that Serena will win the game?

Picture It

Find the theoretical probability.

Describe the sample space.

The sample space is the set of possible numbers that can be rolled: 1, 2, 3, 4, 5, and 6.

Each outcome is equally likely.

The favorable outcome is the number Serena needs to win: the number 4.

$$P(4) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{6}$$

Model It

Find the experimental probability.

Find the total number of rolls.

$$3 + 6 + 2 + 5 + 4 + 4 = 24$$

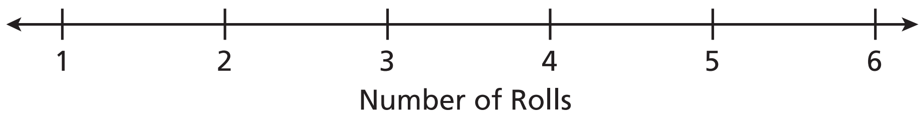
The favorable outcome is the number of times 4 was rolled: 5 times

$$P(4) = \frac{\text{number of times 4 was rolled}}{\text{total number of rolls}} = \frac{5}{24}$$

Connect It

Now you will solve the problem from the previous page using probability.

9 Repeat the experiment using a number cube. Roll the number cube 24 times. Record your results on the line plot.



10 Based on your results, what is the experimental probability that Serena will win the game?

____

11 Compare your results to the given experimental data. How are your results similar? How are they different? ____

____

12 Combine the results of your experiment with those from your class. Record the results in the table to the right.

13 Based on the class results, what is the experimental probability that Serena will win?

____

Number	Number of Rolls
1	
2	
3	
4	
5	
6	

Try It

Use what you learned about experimental probability to answer these questions.

14 Compare the experimental probability of the combined data from your class to the theoretical probability that Serena will win. ______________________________

15 Why might the combined data be a better prediction of the probability than your results alone?

____

____

Read the problem below. Then explore different ways to understand it.

Rafael tosses a paper cup and records how the paper cup lands. The table shows the results of his experiment.

Paper Cup Landing	Number of Times
Sideways	16
Open-end down	4
Open-end up	0

Predict whether the paper cup will land open-end down on his next toss. Use probability to explain your prediction.

Model It

Write the experimental probability of each landing.

Find the total number of trials.

$16 + 4 + 0 = 20$

$$P(\text{Sideways}) = \frac{\text{number of sideways landings}}{\text{number of trials}} = \frac{16}{20} = \frac{4}{5}$$

$$P(\text{Open-end down}) = \frac{\text{number of open-end down landings}}{\text{number of trials}} = \frac{4}{20} = \frac{1}{5}$$

$$P(\text{Open-end up}) = \frac{\text{number of open-end up landings}}{\text{number of trials}} = \frac{0}{20} = 0$$

Picture It

Understand the experiment.

The paper cup lands sideways 16 times, open-end down 4 times, and never open-end up.

The paper cup never lands with the open-end up. The top of the cup is open while the bottom is closed. So, the weights of the ends are different.

The experiment does not have equally likely outcomes.

You cannot write the theoretical probability without more information.

Use the experimental probability of each landing to make the prediction.

Connect It

Now you will solve the problem from the previous page using probability.

16 Why does it appear that the outcomes for the experiment are not equally likely?

17 Use a paper cup to repeat Rafael's experiment. Record your results for 20 tosses.

Paper Cup Landing	Tally	Number of Times
Sideways		
Open-end down		
Open-end up		

18 Combine the results of your experiment with those from your class. Record the results.

Paper Cup Landing	Tally	Number of Times
Sideways		
Open-end down		
Open-end up		

19 Use the class results to find the experimental probability of each landing.

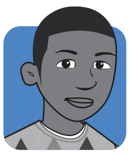
20 The paper cup experiment is non-uniform. Spinning a penny is an experiment that is uniform. Describe the differences and similarities between finding the probability of a paper cup to finding the probability of spinning a penny.

Try It

Use what you learned to solve this problem.

21 How is predicting the paper cup results similar to spinning a penny? How is it different?

There are many different ways to solve this problem. For example, you could let numbers from 00 through 29 represent thin crust and numbers from 30 through 99 represent thick crust.



Study the model below. Then solve problems 22–24.

Student Model

Pablo works at a pizza parlor. On average, 70% of the pizzas ordered have thick crust, and 30% have thin crust. Use the random number table below to find the experimental probability that the next pizza ordered will have thick crust.

75	64	26	45	10	79	18	58	61	09
24	05	89	42	27	98	62	31	19	95
63	18	80	72	41	26	11	91	96	81
38	81	93	68	22	84	92	59	82	80
25	59	54	43	02	16	41	97	40	65

Look at how you could show your work using the random number table.

Let numbers from 00 through 69 represent ordering thick crust.
Let numbers from 70 through 99 represent ordering thin crust.

33 of the 50 numbers are numbers from 00 through 69.

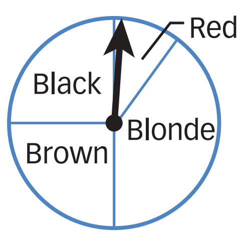
Solution: $\frac{33}{50}$

Pair/Share

How else could you model this problem?

What does the denominator of the fraction represent?

22 Of the 20 students in Kendra's class, 5 have black hair, 8 have blonde hair, 5 have brown hair, and 2 have red hair. Kendra used the spinner to find the experimental probability that this week's class leader will have blonde hair. She spun the spinner 15 times and recorded her results.



Color	Number of Times
Black	2
Blonde	9
Brown	3
Red	1

What is the experimental probability that this week's class leader will have blonde hair?

Show your work.

Solution: ______________________

Pair/Share

Why are the sections of the spinner different sizes?

23 Staci answers a 6-question true or false quiz randomly. What is the experimental probability that Staci will correctly answer exactly half of the questions?

Model the problem by rolling six number cubes at the same time. Let even numbers represent a correct answer and odd numbers represent an incorrect answer. Roll the number cubes 9 times and record your results in the table. The first trial has been done for you.

Trial	Outcomes	Trial	Outcomes	Trial	Outcomes
1	1, 5, 4, 5, 3, 3	4		7	
2		5		8	
3		6		9	

Show your work.

Solution: ______________________

Each of the 6 numbers in the outcomes represents 1 quiz question.

Pair/Share

What can you use instead of a number cube to model this situation?

24 On average, Ava makes 80% of her free throws. In the following random number table, any number from 0 through 7 represents a make, and an 8 or 9 represents a miss. Start at the top left of the table and look at 20 consecutive number pairs as you move to the right to represent Ava's next two free throws. For example, a number pair of 09 represents making the first free throw and missing the second. Based on the model, what is the experimental probability that Ava will make her next two free throws?

23894	55887	76938	66418	19267	59483	79445	30244
71015	49587	24440	05358	95457	78735	18544	39789
94730	89266	57662	16391	51709	06348	48464	53014

A $\frac{1}{2}$

B $\frac{11}{20}$

C $\frac{13}{20}$

D $\frac{4}{5}$

Bill chose **D** as the correct answer. How did Bill get that answer?

____

____

____

Even though there's a gap between the 5th and 6th numbers, they still represent a number pair.

Pair/Share

Does Bill's answer make sense?