Discrete Random Variables

- Topics: Expected Value

- Objective: Students will be able to calculate expected value given a probability distribution.

- Standards: CCSS Math: HSS.MD.A.2, HSS.MD.B.5, HSS.MD.B.5a

Expected Value

Definition: The ***Expected Value*** is the mean of a random variable. A quantity equal to the average result of an experiment after a large number of trials.

Example: What are the outcomes and the corresponding probabilities of rolling a six-sided die?

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Expected Value

Example 1: Given a Table

X is a discrete random variable. The table below defines a probability distribution for X.

What is the expected value of X?

outcomes ← game ← outcome

x	$P(X = x)$
2	0.06
3	0.42
7	0.52

6%

42%

52%

2 * 0.06 = .12

3 * 0.42 = 1.26

7 * 0.52 = 3.64

} add together

5.02 Expected value

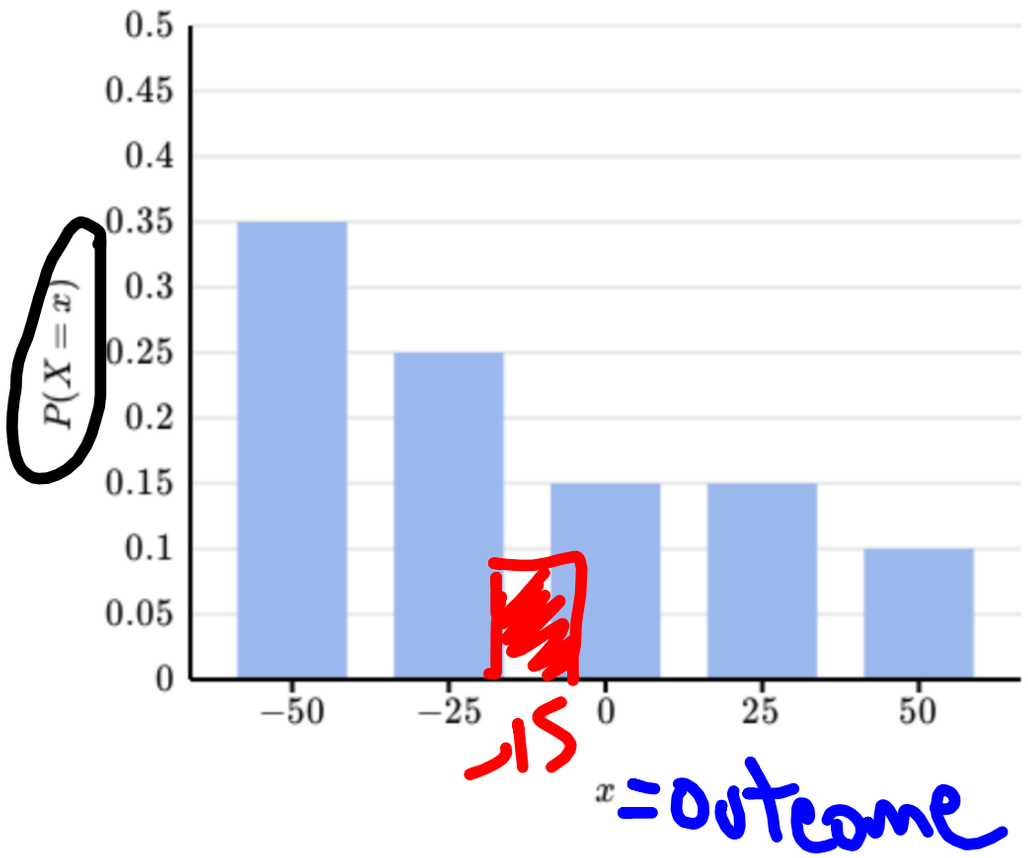
5.02

Do Not Round Answers!

Expected Value

Example 2: Given a Chart

X is a discrete random variable. The graph below defines a probability distribution for X.
What is the expected value of X?



x	$P(X=x)$	$x \cdot P(X=x)$
-50	.35	-17.5
-25	.25	-6.25
0	.15	0
25	.15	3.75
50	.1	5

Add

-15
E.V

Expected Value

Example 3: Given a Story, Need to create a Sample Space, and Answer Questions

Joseph just bought a brand new cell phone and a warranty for the cell phone. The warranty cost $300 and is worth $800 if his phone breaks. Joseph estimates that there is a 20% chance of his phone breaking.

Joseph calculates the expected value of buying the warranty to be -$140.

Which of the following statements are true?

1. Joseph will certainly lose value from the warranty.

2. Joseph will certainly gain value from the warranty.

3. If Joseph was going to buy a large number of phones, and he bought the warranty on each phone, he should expect to lose value from the warranties.

Expected Value

Example 4: Given a Story, Need to create a Sample Space

Outcomes

Mahnoor owns and operates Mahnoor's Coffee Shop. The city of Laketown, Australia, where Mahnoor's Coffee Shop is located, recently enacted a ban on all foam cups to help protect the environment.

positive cost

Instead of switching to paper cups, Mahnoor has decided to risk being fined by the city and to continue to use foam cups. She estimates that this will save her 10,000 Australian dollars. She also estimates that there is a 12% chance that she will be fined. The fine would be for 100,000 Australian dollars.

Cost ↑ losing money = negative

Find the expected value of Mahnoor's decision to continue to use foam cups.

Outcomes: Fined Not Fined

Costs 10,000 Add 10000(.88)

$-100,000$

$-\$0,000(.12) = -10800 + 8800$

$-\$2000$

100%
-12%
88%

Expected Value

Example 4: Given a Story, Need to create a Sample Space

Outcomes

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Instead of switching to paper cups, Mahnoor has decided to risk being fined by the city and to continue to use foam cups. She estimates that this will save her 10,000 Australian dollars. She also estimates that there is a 12% chance that she will be fined. The fine would be for 100,000 Australian dollars.

Cost ↑ losing money = negative

Find the expected value of Mahnoor's decision to continue to use foam cups.

Outcomes: Fined | Not Fined

Costs: 10,000 | 10000(.88)

− 100,000 | × P(X=x)

−90,000 | −90,000 * .12 = −10,800

| 10,000 * .88 = 8800

−2,000 EV

Expected Value

Example 5: Given a Story, Need to create a Sample Space

Outcomes

Marciel is considering buying a flood insurance policy for the upcoming year that costs $600 and is worth $50,000 if her home floods.

− +

Based on previous years, she estimates that there is a 2% chance of her home flooding this year.

Find the expected value of buying this flood insurance policy.

Outcomes	Costs	P(outcome)
Flood	(−600 + 50,000) = 49,400	.02 = 988
Not Flood	−600	.98 = −588

* * Add

$400 E.V.

You should be working on the following skills:

1. Expected Value