Unit 5

Exponential Functions

\[ y = 2^x \]
\[ y = 3^x \]
\[ y = 4^x \]

Asymptote

Created by: M. Signore & G. Garcia
Lesson #38: Zero & Negative Exponents

Do Now:

Zero Exponents

**Rule #1:**

An Exponent of ZERO

\[ a^0 = \]

\[ 3^0 = (-3)^0 = x^0 = \]

a) \((-6)^0 = \]

b) \(x^0 = \]

c) \(-(7)^0 = \]

d) \((25xyz)^0 = \]

e) \(-y^0 = \]

f) word\(^0 = \]

**REMEMBER:** anything to the zero power is 1

\[ 15^0 = 1 \]

\[-(8x)^0 = -1 \Rightarrow (8x)^0 = 1 \text{ but the } - \text{ sign is not raised to the 0 power} \]

\[ \left(\frac{4}{3}\right)^0 = 1 \]
### Negative Exponents

#### CAUTION


A negative exponent does NOT make the number negative!!!

Ex: _________________

Instead, a negative exponent says:

“I’m feeling negative b/c I do not like where I _____ right now. I need to make a change and _____ to a new a new place so I can feel _____________ again.”

#### RULE:

$$10^{-2} = \frac{1}{10^2}$$

All negative exponents can be written as a fraction with 1 in the numerator and the positive exponent in the denominator.

#### Let’s Try:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1. $\frac{1}{6^{-2}}$</td>
<td>2. $\frac{2}{x^{-2}}$</td>
<td>3. $\frac{y^{-3}}{3}$</td>
</tr>
<tr>
<td>4. $-2^0$</td>
<td>5. $(-3)^2$</td>
<td>6. $\frac{1}{2^{-2}}$</td>
</tr>
<tr>
<td>7. $(\frac{1}{4})^{-2}$</td>
<td>8. $3^{-2}$</td>
<td>9. $(-3x)^2$</td>
</tr>
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<td>---</td>
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<td>---</td>
</tr>
<tr>
<td>1.</td>
<td>$2^4$</td>
<td>2.</td>
</tr>
<tr>
<td>4.</td>
<td>$3z^2$</td>
<td>5.</td>
</tr>
<tr>
<td>7.</td>
<td>$2^3 \cdot 2^3$</td>
<td>8.</td>
</tr>
<tr>
<td>10.</td>
<td>$\frac{x^4}{x^6}$</td>
<td>11.</td>
</tr>
</tbody>
</table>
Homework #38: Zero & Negative Exponents

Directions: Rewrite each item as an equivalent expression in exponential notation.
Answers should only have positive exponents.

1) \( \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = \)
2) \( \frac{(-2)}{(-2)} = \)
3) \( \frac{(0.12)(0.12)(0.12)}{(0.12)(0.12)} = \)
4) \( \frac{7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7} = \)
5) \( \frac{15^6}{15^9} = \)
6) \( \frac{(-7)^5}{(-7)^3} = \)
7) \( \frac{\left( \frac{3}{4} \right)^5}{\left( \frac{3}{4} \right)} = \)
8) \( \frac{12^4}{3^5} = \)
9) \( \frac{6^9}{6^9} = \)
10) \( \frac{7^4 \cdot 7^5}{7^9} = \)
11) \( \frac{(9^3)^0 \cdot 5^2}{5^5} = \)

Write the following algebraic problems in exponential notation.

12) \( \frac{x^7}{x^3} = \)
13) \( \frac{a^2b}{a^6b^2} = \)
Rewrite each item as an equivalent expression in exponential notation. Answers should only have positive exponents.

14) \( \frac{t^5}{t^5} = \)

15) \( \frac{x^4 y^2}{x^3 y^8} = \)

Tell whether each statement is correct. Show work to support your answer.

16) \( 2^{-5} = \)

17) \( (-6)^{-4} = \)

18) \( (-5)^{-3} = \frac{1}{(-5)^{-3}} \)

19) \( \frac{8^4}{8^4} = 8 \)

20) \( 7^0 = \frac{7^5}{7^5} \)

21) \( \frac{x^8}{x^4} = x^2 \)

22) \( 5^6 \cdot \frac{1}{25} = 5^8 \)

23) \( (7^4)^{-2} = \frac{1}{49^2} \)
Lesson #39: Intro to Exponential Functions

Do Now:

Simplify the Following:

“Negative Exponents are Bad Manners in Math!”

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>((-4)^2)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(-4^2)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(3^2 \cdot (-2)^3)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(a^2 \cdot a^3)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>((d^4)(d^6))</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>((x^4)^3)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>((2x^2y^3)^4)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(z^0 \cdot z^2)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(x^{-3})</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(5^{-2})</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>((-2x^2)(6x^3)(x^2))</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(\frac{12a^{-2}}{4})</td>
<td></td>
</tr>
</tbody>
</table>
An exponential function with base \( b \) is defined by
\[ f(x) = ab^x \]
where \( a \neq 0 \), \( b > 0 \), \( b \neq 1 \), and \( x \) is any real number.
The base, \( b \), is constant and the exponent, \( x \), is a variable.

In the following example, \( a = 1 \) and \( b = 2 \).

\[
\begin{array}{c|c}
  x & y = f(x) \\
  \hline
  -2 & 2^{-2} = \frac{1}{4} \\
  -1 & 2^{-1} = \frac{1}{2} \\
  0 & 2^0 = 1 \\
  1 & 2^1 = 2 \\
  2 & 2^2 = 4 \\
  3 & 2^3 = 8 \\
\end{array}
\]

**Shape:** Most exponential graphs will have this same arcing shape.

**Rate of Change:**
This graph does not have a constant rate of change, but it has constant ratios. It is growing by common factors over equal intervals.

**Features** (for this graph):
- the **domain** is all Real numbers.
- the **range** is all positive real numbers (not zero).
- graph has a **y-intercept** at (0,1). Remember any number to the zero power is 1.
- when \( b > 1 \), the graph increases. The greater the base, \( b \), the faster the graph rises from left to right.
- when \( 0 < b < 1 \), the graph decreases.
- has an **asymptote** (a line that the graph gets very, very close to, but never crosses or touches). For this graph the asymptote is the x-axis (\( y = 0 \)).
Graphing Exponential Functions:

1] \( f(x) = 2^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2] \( g(x) = 3^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Observations:
3) \( f(x) = 0.5^x \)

\[
egin{array}{c|c}
 x & y \\
-1 & \\
0 & \\
1 & \\
2 & \\
\end{array}
\]

4) \( h(x) = -2^x \)

\[
egin{array}{c|c}
 x & y \\
-1 & \\
0 & \\
1 & \\
2 & \\
\end{array}
\]
Graph the functions

1. \( f(x) = 4^x \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

2. \( f(x) = 1.25^x \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
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<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Determine the equation of each exponential function below

**PROCESS**

1) Locate 3-4 definite points on the graph, circle and name them.

2) On your calculator, press STAT - EDIT

3) Enter all “x” values into L1 and all “y” values into L2

4) Press STAT - CALC - choose 0:ExpReg. Hit enter all the way through until the screen changes

5) Copy the equation onto your paper and substitute the values of “a” and “b”
Determine the equation of each exponential function below.
Lesson #40: Exponential Growth and Decay

Do Now:

Jump Start your Prior Knowledge

1] Given \( y = 3^x \), evaluate \( y \) when \( x = 3 \). 

2] Given \( y = 3^x \), evaluate \( y \) when \( x = -2 \).

3] Which ordered pair represents the \( y \)-intercept for the function \( y = 2^x \)?
   a) (0,0)           b) (0, 1)           c) (0, 2)

4] The graph of \( y = 2^x \) lies in which Quadrants?
   a) I, II           b) I, III           c) I, IV

5] The graph of \( y = 2^x \) contains which of these points?
   a) (0,0)           b) (0, 1)           c) (0, 2)
# Exponential Functions

$$y = a \cdot b^x$$

## Exponential Growth

Graph: $y = 2^x$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

## Exponential Decay

Graph: $y = 0.5^x$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Growth: $b > 1$

Decay: $0 \leq b \leq 1$
**Exponential Growth vs. Decay:**

**Example:**
Would the graph of $y = 0.5^x$ show exponential growth or exponential decay?

**Example:**
Would the graph of $y = 1.5^x$ show exponential growth or exponential decay?

**Exponentials in the Real World?**
Many real world phenomena can be modeled by functions that describe how things grow or decay as time passes. Examples of such phenomena include the studies of populations, bacteria, the AIDS virus, radioactive substances, electricity, temperatures and credit payments.

<table>
<thead>
<tr>
<th>Exponential Growth:</th>
<th>Exponential Decay:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = a(1 + r)^x$</td>
<td>$y = a(1 - r)^x$</td>
</tr>
</tbody>
</table>

**Ingredients:**
- $a =$ initial **amount** before measuring growth/decay
- $r =$ growth/decay **rate** (often a percent)

⚠️ **IMPORTANT**→ **convert** to a decimal
- $x =$ number of **time** intervals that have passed (years)

**Growth Example:**
A bank account balance, $b$, for an account starting with $s$ dollars, earning an annual interest rate, $r$, and left untouched for $n$ years can be calculated as $b = s(1 + r)^n$ (an exponential growth formula). Find a bank account balance to the nearest dollar, if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

Decay Example:
Daniel's Print Shop purchased a new printer for $35,000. Each year it depreciates (loses value) at a rate of 5%. What will its approximate value be at the end of the fourth year?

A) $33,250.00    B) $30,008.13    C) $28,507.72    D) $27,082.33

U-Try:

1] Cassandra bought an antique dresser for $500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the nearest dollar?
2) The value, \( y \), of a $15,000 investment over \( x \) years is represented by the equation, 
\[ y = 15000(1.2)^{\frac{x}{3}} \]. What is the profit (interest) on a 6-year investment?
1) $6,600
2) $10,799
3) $21,600
4) $25,799

3) The New York Volleyball Association invited 64 teams to compete in a tournament. After each round, half of the teams were eliminated. Which equation represents the number of teams, \( t \), that remained in the tournament after \( r \) rounds?
1) \( t = 64(r^{0.5}) \)
2) \( t = 64(-0.5)^r \)
3) \( t = 64(1.5)^r \)
4) \( t = 64(0.5)^r \)

4) In a science fiction novel, the main character found a mysterious rock that decreased in size each day. The table below shows the part of the rock that remained at noon on successive days.

<table>
<thead>
<tr>
<th>Day</th>
<th>Fractional Part of the Rock Remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{8} )</td>
</tr>
</tbody>
</table>

Which fractional part of the rock will remain at noon on day 7?
1) \( \frac{1}{128} \) 3) \( \frac{1}{12} \)
2) \( \frac{1}{64} \) 4) \( \frac{1}{14} \)
For each word problem, write the exponential equation to model the situation.

4) A zombie infection in Yonkers Public Schools grows by 15% per hour. The initial group of zombies was a group of 4 freshmen. How many zombies are there after 6 hours.
5) Ryan is saving for his college tuition. He has $2,550 in a savings account that pays 6.25% annual interest.

6) Cars depreciate in value over time. A used car was purchased for $12,329 this year. Each year the car’s value decreases 8.5%.

7) Jeremiah owns a side business detailing cars. His first year he made $10,500 and each of the following years his profit increased 9%.

8) There are 128 teams entered in a basketball tournament. Half of the teams are eliminated each round. How many teams are left after 4 rounds?

9) Bacteria in a dirty glass triple every year. If there are 25 bacteria to start, how many are in the glass after 1 day?

10) The population of a city with 750,000 is devastated by an unknown virus that kills 20% of the population per day. How many people are left after a week?
11) There are 1,750,235 acres of forest in northwestern Idaho. One-half percent of the forest is destroyed by pollution every year. How many acres are left after 65 years?

12) A new iPhone is estimated to lose 25% of its value every six months after its purchase. How much is the value of an iPhone that costs $799 after someone has owned it for 2 years?

13) A recent college grad accepts a job at Google Inc. The job has a salary of $50,000 and is guaranteed an annual pay increase of 3%. What will their salary be after 7 years?
Lesson #41: Modeling with Exponential Functions

Recap:

Parts of an Exponential Function

The general equation for an exponential GROWTH function is:
\[ y = a(b)^x \]
where,
- \( a \): initial value
- \( b \geq 1 \): growth
- \( b > 1 \) then \( b \): growth FACTOR
- \( b = 1 + r \), \( r = b - 1 \)

The general equation for an exponential DECAY function is:
\[ y = a(b)^x \]
where,
- \( a \): initial value
- \( 0 < b < 1 \): decay
- \( b = 1 - r \), \( r = 1 - b \)

Do Now: Identify the initial value, the growth or decay factor, and the growth or decay rate of the following exponential functions.

1. \( y = 3(1.8)^x \)
   - Growth or decay
   - Initial Value
   - Growth or decay factor
   - Growth or decay rate

2. \( y = 2.1(1.04)^x \)
   - Growth or decay
   - Initial Value
   - Growth or decay factor
   - Growth or decay rate
3. The Johnson Company calculates the value of its stock each year by using the function \( y = 120 \cdot (0.98)^x \).

<table>
<thead>
<tr>
<th>Growth or decay</th>
<th>Growth or decay factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Value</td>
<td>Growth or decay rate</td>
</tr>
</tbody>
</table>

**Word Problems Relating to Exponential Growth and Decay**

1. A house that costs $200,000 will appreciate in value by 3% each year.

   Write a function that models the cost of the house over time. Use \( x \) for years and \( y \) for the value of the house, in dollars.

   Find the value of the house at the end of ten years.

2. The most recent virus that is making people ill is a fast multiplying one. On the first day of the illness, only 2 virus “bugs” are present. Each day after, the amount of “bugs” triples.

   Write a function that models the “bugs” growth over time. Use \( x \) for days and \( y \) for the amount of “bugs.”

   Find the amount of “bugs” present by the 5th day.

3. Toby ate half a banana in his room and forgot to throw the rest away. That night, two gnats came to visit the banana. Each night after, there were four times as many gnats hanging around the banana.

   Write a function that models the gnats’ growth over time. Use \( x \) for the nights and \( y \) for the number of gnats.
Tobys mom said that he will be grounded if the gnats number more than 120. On what night will Toby be in trouble if he doesn’t step in and solve the gnat problem?

4. You have a bad cough and have to attend your little sister’s concert. You take cough drops that contain 100mg of menthol in each drop. Every minute, the amount of menthol in your body is cut in half.
Write a function that models the amount of menthol in your body over time. Use x for minutes and y for the amount of menthol, in mg, remaining in your body.

5. Ian’s new Mercedes cost him $75,000. From the moment he drives it off the lot, it will depreciate by 20% each year for the first five years.
Write a function that models the car’s depreciation. Use x for years and y for the car’s value, in dollars.
What will the car’s value be at the end of five years?

6. Find a bank account balance if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

7. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person’s system decreases by about 29%. How much ibuprofen is left after 6 hours?
Exponential Growth and Decay Extra Practice

Problems

Make a table of values and draw a graph of each exponential function.

1. \( y = 4(0.5)^x \)  
2. \( y = 2(3)^x \)  
3. \( y = 5(1.2)^x \)  
4. \( y = 10(\frac{2}{3})^x \)

<table>
<thead>
<tr>
<th>Table of Values</th>
<th>Sketch of Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
</tr>
</tbody>
</table>
5. The number of bacteria present in a colony is 180 at 12 noon and the bacteria grows at a rate of 22% per hour. How many will be present at 8 p.m.?

6. A house purchased for $226,000 has lost 4% of its value each year for the past five years. What is it worth now?
7. A 1970 comic book has appreciated 10% per year and originally sold for $0.35. What will it be worth in 2010?

8. A Honda Accord depreciates at 15% per year. Six year ago it was purchased for $21,000. What is it worth now?

9. Inflation is at a rate of 7% per year. Today Janelle's favorite bread costs $3.79. What would it have cost ten years ago?

10. Ryan's motorcycle is now worth $2500. It has decreased in value 12% each year since it was purchased. If he bought it four years ago, what did it cost new?

11. The cost of a High Definition television now averages $1200, but the cost is decreasing about 15% per year. In how many years will the cost be under $500?
12. A two-bedroom house in Nashville is worth $110,000. If it appreciates at 2.5% per year, when will it be worth $200,000?

13. Last year the principal's car was worth $28,000. Next year it will be worth $25,270. What is the annual rate of depreciation? What is the car worth now?
Lesson #41A: Compound Interest & Percent of Change

**Compound interest**: Interest that is earned on both the principal and any gains made in previous years.

**Compound interest formula:**

\[ A = P (1 + r)^t \]

- \( a \) = amount at end of term
- \( p \) = initial investment/principal
- \( r \) = interest rate
- \( t \) = time

---

A) A $1250 invested at 8% compounded for 2 years
B) $650 invested at 7% compounded annually for 5 years

C) $10,000 invested at 7.8% compounded annually for 2 years
D) $7,500 invested at 6% compounded annually for 15 yrs
**Percent of Change**

\[
\text{new - original} \times \frac{100}{\text{original}}
\]

**Percent of Change** - the percent by which a number increases or decreases

**percent decrease** - a percent change describing a decrease in a quantity

**percent increase** - the percent change describing an increase in a quantity

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**Guided Practice**

Find each percent increase. Round to the nearest percent. (Example 1)

1. From $5 to $8  
2. From 20 students to 30 students 
3. From 86 books to 150 books  
4. From $3.49 to $3.89 
5. From 13 friends to 14 friends  
6. From 5 miles to 16 miles 
7. Nathan usually drinks 36 ounces of water per day. He read that he should drink 64 ounces of water per day. If he starts drinking 64 ounces, what is the percent increase? Round to the nearest percent. (Example 1) 

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Find each percent decrease. Round to the nearest percent. (Example 2)

8. From $80 to $64  
9. From 95°F to 68°F 
10. From 90 points to 45 points  
11. From 145 pounds to 132 pounds 
12. From 64 photos to 21 photos  
13. From 16 bagels to 0 bagels 
14. Over the summer, Jackie played video games 3 hours per day. When school began in the fall, she was only allowed to play video games for half an hour per day. What is the percent decrease? Round to the nearest percent. (Example 2)
Percent of Change Word Problems

1. If the price of a purse increased from $45 to $54, what was the percent of increase?

2. If the price of a softball glove decreased from $60 to $36, what was the percent of decrease?

3. Mary decreased her time in the mile walk from 30 minutes to 24 minutes. What was the percent of decrease?

4. A DVD movie originally cost $24.99. Its current price is $19.99. What is the percent of change rounded to the nearest percent?
**HW #41A: Compound Interest & Percent of Change**

**Directions:** State whether each percent of change is a percent increase or a percent decrease. Then find the percent of increase or decrease. Round to the nearest whole percent.

1. Original: $100  
   New: $59

2. Original: 324 people  
   New: 549 people

3. Original: 58 Homes  
   New: 152 Homes

4. Original: 66 Dimes  
   New: 30 Dimes

5. Original: $53  
   New: $75

6. Original: 15.6 liters  
   New: 11.4 liters

7. Original: $3.78  
   New: $2.50

8. Original: 231.2 mph  
   New: 236.4 mph
Lesson #42: Linear Regressions

Recap: Graphing Linear Functions
1. Graph the linear equation by finding the x- and y-intercepts.
   \(-3x + 2y = -12\)
   (SHOW WORK using algebra!)
2. Rewrite the equation into slope-intercept form. Graph using the slope and the y-intercept.
   \(x - 4y = 8\)

2.5: Fitting a Line to Data (Linear Regression) ~ Table to Equation

We know that, in general, a line with a positive slope looks like this.

A scatter plot is the graph of a set of _________. Correlation means how the points in the scatter plot are related to each other. Here are some examples:

- Correlation
- Correlation
- Correlation

Example: Determine the Correlation of a Scatter Plot

1.
2.
3.

- Correlation
- Correlation
- Correlation

Linear Regression

Sometimes there is no single line that passes through all of the data points, so you try to find the line that best fits the data. Fitting a line to data is called finding the line of ____________________ or linear ____________________.
To create a scatter plot, calculate and draw regression equation:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>-3</td>
<td>-2</td>
<td>5</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

1. To enter the set of data in a list:
   - Press `STAT`; 1
   - If there is already data in the list, arrow up and highlight L1, press `CLEAR` and arrow once down.
   - Enter the data into L1 (x-values).
   - Enter the data into L2 (y-values).

2. To create a scatter plot:
   - Enter data into a list.
   - Press `Y=` and clear any equations.
   - Press `2nd`, `Y=`, 1
   - Highlight `ON` and press `ENTER`.
   - Arrow down and select type: scatter plot.
   - Xlist should read L1 and Ylist should read L2.
   - Press `ZOOM` 9 (ZOOM Stat).

3. To calculate the line of best fit:
   - Press `STAT`.
   - Arrow over to `CALC` and select 4(`LinReg (ax+b)`).

   The line of regression is: ________________________________

4. To graph the line of best fit:
   - Press `Y=`.
   - Type the Line of Regression from #3.
   - Press `GRAPH`.

Writing an Regression Equation from a Table. (Round to 3 decimal places)

1. | x   | 1.0 | 1.5 | 1.7 | 2.0 | 2.0 | 1.5 |
   | y   | 3.8 | 4.2 | 5.3 | 5.8 | 5.5 | 6.7 |

   Use the regression equation, predict y when x = 4.

   Regression Equation: ________________________________
   Type of Correlation: ________________________________

2. | x   | 3.0 | 3.5 | 3.7 | 4.0 | 4.0 | 4.5 |
   | y   | 9.9 | 9.7 | 8.6 | 8.1 | 8.4 | 7.4 |

   Use the regression equation:
   a. predict y when x = 7.
   b. find x when y = 15.

   Regression Equation: ________________________________
   Type of Correlation: ________________________________
3. Use the regression equation:
   a. predict $y$ when $x = 10$.
   b. find $x$ when $y = 15$.

4. Use the regression equation:
   a. predict $y$ when $x = -3.1$.
   b. find $x$ when $y = 10$.

5. Crickets are known to chirp faster at higher temperature and slower at lower temperatures. The number of chirps is thus a function of the temperature. The following data were collected and recorded in a table.

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of chirps per minute</td>
<td>11</td>
<td>29</td>
<td>47</td>
<td>75</td>
<td>109</td>
</tr>
</tbody>
</table>

   a. Graph the data with temperature on the x-axis and chirps per minutes on the y-axis (make a scatter plot).
   b. Draw the line of best fit using a straight edge.
   c. Select 2 points on or close to the line.
   d. State the x and the y coordinates of these points.
   e. Find the slope of the line from these two points (subtract the points).
   f. Find the equation of the line of best fit using the point-slope formula:
      $$y - y_1 = m(x - x_1)$$
   g. Find the line of regression using the graphics calculator.
   h. According to this model, what do you predict to be the number of chirps per minute when the temperature is 18°C? When the temperature is 25°C?
Writing an Regression Equation for a Real-Life Model

1. The table shows the average number of gallons of milk a family drinks per week.

<table>
<thead>
<tr>
<th>Family Size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Gallons</td>
<td>1</td>
<td>1.5</td>
<td>2.2</td>
<td>3.8</td>
<td>4.7</td>
<td>5</td>
</tr>
</tbody>
</table>

a. Find the regression equation.  
b. Find the milk consumption in one week of a 7-member family.

2. The table shows the number of dollars (in billions) spent on toys and sports supplies in the United States from 1990 through 1995.

<table>
<thead>
<tr>
<th>Years Since 1990</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billions of Dollars</td>
<td>31.6</td>
<td>32.8</td>
<td>?</td>
<td>36.5</td>
<td>40.1</td>
<td>42.7</td>
</tr>
</tbody>
</table>

a. Find the regression equation.  
b. Estimate the amount spent on toys and sports supplies in 1992.

3. FUEL ECONOMY The table gives the weights in tons and estimates the fuel economy in miles per gallon for several cars.

<table>
<thead>
<tr>
<th>Weight (tons)</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.8</th>
<th>2</th>
<th>2.1</th>
<th>2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles per Gallon</td>
<td>29</td>
<td>24</td>
<td>23</td>
<td>21</td>
<td>?</td>
<td>17</td>
<td>15</td>
</tr>
</tbody>
</table>

a. Make a scatter plot using the data.  
b. Draw a line of best fit.  
c. Enter the data into the calculator and find the line of regression. (round to 3 decimal places).
4. The table below shows the number of grams of carbohydrates, \( x \), and the number of Calories, \( y \), of six different foods.

<table>
<thead>
<tr>
<th>Carbohydrates (( x ))</th>
<th>Calories (( y ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>120</td>
</tr>
<tr>
<td>9.5</td>
<td>138</td>
</tr>
<tr>
<td>10</td>
<td>147</td>
</tr>
<tr>
<td>6</td>
<td>88</td>
</tr>
<tr>
<td>7</td>
<td>108</td>
</tr>
<tr>
<td>4</td>
<td>62</td>
</tr>
</tbody>
</table>

Which equation best represents the line of best fit for this set of data?

a. \( y = 15x \)  
b. \( y = 0.07x \)  
c. \( y = 0.1x - 0.4 \)  
d. \( y = 14.1x + 5.8 \)

5. The table below shows the number of grams of carbohydrates, \( x \), and the number of Calories, \( y \), of six different foods.

<table>
<thead>
<tr>
<th>Carbohydrates (( x ))</th>
<th>Calories (( y ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>120</td>
</tr>
<tr>
<td>9.5</td>
<td>138</td>
</tr>
<tr>
<td>10</td>
<td>147</td>
</tr>
<tr>
<td>6</td>
<td>88</td>
</tr>
<tr>
<td>7</td>
<td>108</td>
</tr>
<tr>
<td>4</td>
<td>62</td>
</tr>
</tbody>
</table>

Which equation best represents the line of best fit for this set of data?

a. \( y = 15x \)  
b. \( y = 0.07x \)  
c. \( y = 0.1x - 0.4 \)  
d. \( y = 14.1x + 5.8 \)

6. The table below shows the attendance at a museum in select years from 2007 to 2013.

<table>
<thead>
<tr>
<th>Year</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2011</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance (millions)</td>
<td>8.3</td>
<td>8.5</td>
<td>8.5</td>
<td>8.8</td>
<td>9.3</td>
</tr>
</tbody>
</table>

State the linear regression equation represented by the data table when \( x = 0 \) is used to represent the year 2007 and \( y \) is used to represent the attendance. Round all values to the nearest hundredth. State the correlation coefficient to the nearest hundredth and determine whether the data suggest a strong or weak association.
Emma recently purchased a new car. She decided to keep track of how many gallons of gas she used on five of her business trips. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Miles Driven</th>
<th>Number of Gallons Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>7</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>19</td>
</tr>
<tr>
<td>600</td>
<td>29</td>
</tr>
<tr>
<td>1000</td>
<td>51</td>
</tr>
</tbody>
</table>

Write the linear regression equation for these data where miles driven is the independent variable. (Round all values to the nearest hundredth.)

8.

Rachel and Marc were given the information shown below about the bacteria growing in a Petri dish in biology class.

<table>
<thead>
<tr>
<th>Number of Hours, x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bacteria, B(x)</td>
<td>220</td>
<td>280</td>
<td>350</td>
<td>440</td>
<td>550</td>
<td>690</td>
<td>860</td>
<td>1070</td>
<td>1340</td>
<td>1680</td>
</tr>
</tbody>
</table>

Rachel wants to model this information with a linear function. Marc wants to use an exponential function model is the better choice? Explain why you chose this model.

9.

Each day Toni records the height of a plant for her science lab. Her data are shown in the table below.

<table>
<thead>
<tr>
<th>Day (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>3.0</td>
<td>4.5</td>
<td>6.0</td>
<td>7.5</td>
<td>9.0</td>
</tr>
</tbody>
</table>

The plant continues to grow at a constant daily rate. Write an equation to represent \( h(n) \), the height of the plant on the \( n \)th day.
Lesson #42A: Calculating the Correlation Coefficient

Vocabulary

**Correlation**: A statistical measure that quantifies how pairs of variables are related; a linear relationship between two variables.

**Correlation Coefficient**: A number between -1 and 1 that indicates the strength and direction of the linear relationship between two sets of numbers. The letter “r” is used to represent correlation coefficients.

**Interpreting a Correlation Coefficient - What It Means**

Every correlation coefficient has two pieces of information:

1. **The sign of the correlation**: A correlation is either positive or negative.
   a. With positive correlations, the variables increase or decrease together.
   b. With negative correlations, one variable increases while the other decreases.

2. **The absolute value of the correlation**.
   a. The closer the absolute value of the correlation is to 1, the stronger the correlation between the variables.
   b. The closer the absolute value of the correlation is to zero, the weaker the correlation between the variables.
The **sign of the correlation** tells you what the graph will look like and

<table>
<thead>
<tr>
<th>Negative Correlation</th>
<th>No Correlation</th>
<th>Positive Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>In general, one set of data decreases as the other set increases.</td>
<td>Sometimes data sets are not related and there is no general trend.</td>
<td>In general, both sets of data increase together.</td>
</tr>
<tr>
<td>An example of a negative correlation between two variables would be the relationship between absentiism from school and class grades. As one variable increases, the other would be expected to decrease.</td>
<td>A correlation of zero does not always mean that there is no relationship between the variables. It could mean that the relationship is not linear. For example, the correlation between points on a circle or a regular polygon would be zero or very close to zero, but the points are very predictably related.</td>
<td>An example of a positive correlation between two variables would be the relationship between studying for an examination and class grades. As one variable increases, the other would also be expected to increase.</td>
</tr>
</tbody>
</table>

The **absolute value of the correlation** tells you the strength of the correlation.

\[
\begin{array}{ccc}
0 & \text{Weak Correlation} & 1 \\
& \text{Absolute value of } r &
\end{array}
\]

In a perfect correlation, when \( r = \pm 1 \), all data points balance the equations and also lie on the graph of the equation.
How to Calculate a Correlation Coefficient Using a Graphing Calculator:

STEP 1. Press STAT EDIT 1:Edit.

STEP 2. Enter bivariate data in the L1 and L2 columns. All the x-values go into L1 column and all the Y values go into L2 column.

STEP 3. Turn the diagnostics on by pressing 2ND CATALOG and scrolling down to DiagnosticsOn. Then, press ENTER ENTER. The screen should respond with the message Done. NOTE: If Diagnostics are turned off, the correlation coefficient will not appear beneath the regression equation.

Step 4. Use STAT CALC and the appropriate type of regression, then ENTER ENTER.

Step 5. The r value that appears at the bottom of the screen is the correlation coefficient.

What is the correlation coefficient of the linear fit of the data shown below, to the nearest hundredth?

![Graph of data points](image)

a. 1.00  

b. 0.93  

c. −0.93  

d. −1.00
A nutritionist collected information about different brands of beef hot dogs. She made a table showing the number of Calories and the amount of sodium in each hot dog.

<table>
<thead>
<tr>
<th>Calories per Beef Hot Dog</th>
<th>Milligrams of Sodium per Beef Hot Dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>186</td>
<td>495</td>
</tr>
<tr>
<td>181</td>
<td>477</td>
</tr>
<tr>
<td>176</td>
<td>425</td>
</tr>
<tr>
<td>149</td>
<td>322</td>
</tr>
<tr>
<td>184</td>
<td>482</td>
</tr>
<tr>
<td>190</td>
<td>587</td>
</tr>
<tr>
<td>168</td>
<td>370</td>
</tr>
<tr>
<td>139</td>
<td>322</td>
</tr>
</tbody>
</table>

a) Write the correlation coefficient for the line of best fit. Round your answer to the nearest hundredth.

b) Explain what the correlation coefficient suggests in the context of this problem.
Megan and Bryce opened a new store called the Donut Pit. Their goal is to reach a profit of $20,000 in their 18th month of business. The table and scatter plot below represent the profit, $P$, in thousands of dollars, that they made during the first 12 months.

<table>
<thead>
<tr>
<th>$t$ (months)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (profit, in thousands of dollars)</td>
<td>3.0</td>
<td>2.5</td>
<td>4.0</td>
<td>5.0</td>
<td>6.5</td>
<td>5.5</td>
<td>7.0</td>
<td>6.0</td>
<td>7.5</td>
<td>7.0</td>
<td>9.0</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Use the statistical features of your calculator to construct a scatter plot and fit a linear function to the data. Calculate and interpret the correlation coefficient.

Use your equation to predict whether Megan and Bryce will reach their goal in the 18th month of their business. *Show your work.*
The gestation time for a type of animal is the typical time between conception and birth for that type of animal. The longevity of an animal is the typical length of life for that animal. The gestation times (in days) and the longevities (in years) for 13 types of animals are shown in the table below.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Gestation Time (days)</th>
<th>Longevity (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baboon</td>
<td>187</td>
<td>20</td>
</tr>
<tr>
<td>Black Bear</td>
<td>219</td>
<td>18</td>
</tr>
<tr>
<td>Beaver</td>
<td>105</td>
<td>5</td>
</tr>
<tr>
<td>Bison</td>
<td>285</td>
<td>15</td>
</tr>
<tr>
<td>Cat</td>
<td>63</td>
<td>12</td>
</tr>
<tr>
<td>Chimpanzee</td>
<td>230</td>
<td>20</td>
</tr>
<tr>
<td>Cow</td>
<td>284</td>
<td>15</td>
</tr>
<tr>
<td>Dog</td>
<td>61</td>
<td>12</td>
</tr>
<tr>
<td>Fox (Red)</td>
<td>52</td>
<td>7</td>
</tr>
<tr>
<td>Goat</td>
<td>151</td>
<td>8</td>
</tr>
<tr>
<td>Lion</td>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>Sheep</td>
<td>154</td>
<td>12</td>
</tr>
<tr>
<td>Wolf</td>
<td>63</td>
<td>5</td>
</tr>
</tbody>
</table>

a) Does it look like there is a relationship between gestation time and longevity for the animals?

b) Use the statistical features of your calculator to find the equation of the line of best fit for the data. Calculate and interpret the correlation coefficient.
The table shows the relationship between the time a student spends working out each week and his percent improvement on race times.

<table>
<thead>
<tr>
<th>Hours Spent Working Out</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Improvement</td>
<td>18</td>
<td>18</td>
<td>32</td>
<td>27</td>
<td>31</td>
<td>39</td>
<td>37</td>
</tr>
</tbody>
</table>

a) Make a scatter plot for the data.

```
Percent

\[2 4 6 8 10 12 14 16 18 20 22\]

\[44 \quad 40 \quad 36 \quad 32 \quad 28 \quad 24 \quad 20 \quad 16 \quad 12 \quad 8 \quad 4\]
```

b) Use the statistical features of your calculator to fit a linear function to the data. Calculate and interpret the correlation coefficient (round to the nearest thousandth).

c) Use your equation to predict the number of hours the student would be expected to work out if his percent improvement is 50% (round to the nearest hour).
The relationship of a woman's shoe size and length of a woman's foot, in inches, is given in the accompanying table.

<table>
<thead>
<tr>
<th>Women's Shoe Size</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foot Length (in)</td>
<td>9.00</td>
<td>9.25</td>
<td>9.50</td>
<td>9.75</td>
</tr>
</tbody>
</table>

The linear correlation coefficient for this relationship is

A. 1  B. -1  C. 0.5  D. 0

Which graph represents data used in a linear regression that produces a correlation coefficient closest to -1?

A.  
B.  
C.  
D.  

What could be the approximate value of the correlation coefficient for the accompanying scatter plot?

A. -0.85  B. -0.16  C. 0.21  D. 0.90
There is a negative correlation between the number of hours a student watches television and his or her social studies test score. Which scatter plot below displays this correlation?

A.  
B.  
C.  
D.  

Based on the line of best fit drawn below, which value could be expected for the data in June 2015?

A. 230  
B. 310  
C. 480  
D. 540  

Which value of $r$ represents data with a strong positive linear correlation between two variables?

A. 0.89  
B. 0.34  
C. 1.04  
D. 0.01
Lesson #43: Use Residuals to Assess Fit of a Function

Vocabulary

**Residual**: is the vertical distance between where a regression equation predicts a point will appear on a graph and the actual location of the point on the graph (scatterplot). A residual can also be understood as the difference in predicted and actual y-values (dependent variable values) for a given value of x (the independent variable).

**Residual plot**: is a scatter plot that shows the residuals as points on a vertical axis (y-axis) above corresponding (paired) values of the independent variable on the horizontal axis (x-axis).

**Any pattern in a residual plot suggests that the regression equation is not appropriate for the data**

Big Ideas

- Patterns in residual plots are bad.
- Residual plots with patterns indicate the regression equation is not a good fit.
- Residual plots without patterns indicate the regression equation is a good fit.

The residual plots from two different sets of bivariate data are graphed below.

**Graph A**

**Graph B**

Explain, using evidence from graph A and graph B, which graph indicates that the model for the data is a good fit.
Practice #1 Example

Use the data below to write the regression equation \( y = ax + b \) for the raw test score based on the hours tutored. Round all values to the nearest hundredth.

<table>
<thead>
<tr>
<th>Tutor Hours, ( x )</th>
<th>Raw Test Score</th>
<th>Residual (Actual – Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>37</td>
<td>1.9</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>−6.4</td>
</tr>
<tr>
<td>4</td>
<td>47</td>
<td>−0.7</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
<td>2.0</td>
</tr>
<tr>
<td>6</td>
<td>67</td>
<td>6.6</td>
</tr>
<tr>
<td>7</td>
<td>62</td>
<td>−4.7</td>
</tr>
</tbody>
</table>

Equation: ___________________________

Create a residual plot on the axes below, using the residual scores in the table above.

Based on the residual plot, state whether the equation is a good fit for the data. Justify your answer.
Practice #2

The table below represents the residuals for a line of best fit.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Plot these residuals on the set of axes below.

Using the plot, assess the fit of the line for these residuals and justify your answer.

Practice #3

Which of the following residual plots indicate a good fit for a linear model?

![Residual plots]

Explain why based on what you learned about residuals.
Complete each table using the given linear regression (Round answers to one decimal place). Construct a residual plot.

1. Linear regression equation: \( y = 0.5x \)

<table>
<thead>
<tr>
<th></th>
<th>y</th>
<th>Predicted Value</th>
<th>Residual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
<td>7.5</td>
<td>1.5</td>
</tr>
<tr>
<td>20</td>
<td>7</td>
<td>12.5</td>
<td>-5</td>
</tr>
<tr>
<td>25</td>
<td>13</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Does the residual plot suggest a linear relationship? Explain. Yes, No

**Predicted Value**: Substitute the \( x \)-value into the given equation

**Residual Value**: Subtract the predicted value from the \( y \)-value

\((y \text{ value } - \text{ predicted value})\)

2. Linear Regression equation: \( y = -0.4x + 16.3 \)

<table>
<thead>
<tr>
<th></th>
<th>( y ) (Observed Value)</th>
<th>Predicted Value</th>
<th>Residual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Does the residual plot suggest a linear relationship? Explain.
### 3. Linear Regression equation: $y = 0.5x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$ (Observed Value)</th>
<th>Predicted Value</th>
<th>Residual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>505</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>415</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>360</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>305</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>265</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Does the residual plot suggest a linear relationship? Explain.

---

### Independent Practice:

1. The data given below shows the height at various ages for a group of children:

<table>
<thead>
<tr>
<th>Age (months)</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>76</td>
<td>77.1</td>
<td>78.1</td>
<td>78.3</td>
<td>78.8</td>
<td>79.4</td>
<td>79.9</td>
<td>81.3</td>
<td>81.1</td>
<td>82</td>
<td>82.6</td>
<td>83.5</td>
</tr>
</tbody>
</table>

Given the best fit line as: $y = 0.634x + 64.945$

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Predicted</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>77.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>78.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>78.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>78.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>79.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>79.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>81.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>81.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>82.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>83.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Is there a pattern? Is this a good model?
2. 

<table>
<thead>
<tr>
<th>Volume</th>
<th># of People</th>
<th>Predicted</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>195</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.8</td>
<td>188</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.2</td>
<td>183</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>166</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.8</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>161</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>157</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.6</td>
<td>192</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>156</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. If \( y = 26.732x + 16.226 \), plot the residuals after filling in the table.

b. Based on the residuals plot, is a line a good fit of the data? Explain.

3. Consider the following data: The shoe sizes and heights (in inches) for men.

<table>
<thead>
<tr>
<th>Shoe Size (x)</th>
<th>Height (y)</th>
<th>Predicted Height</th>
<th>Residual (Actual-Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>66.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td>68.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td>67.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.5</td>
<td>70.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>70.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>72.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.5</td>
<td>71.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.5</td>
<td>69.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>71.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.0</td>
<td>73.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.0</td>
<td>74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.5</td>
<td>74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Find the equation for the line of best fit, as well as the correlation coefficient.

b. Is there a pattern? Is the prediction line the best model for the data? How can you tell?
Lesson #44: Arithmetic Sequences

An arithmetic sequence is a sequence in which each term after the first is found by adding a constant, called the common difference $d$, to the previous term.

| 1st term: $a_1$ | 2nd term: $a_2$ | 3rd term: $a_3$ | $n^{th}$ term: $a_n$ |

**EXAMPLE 1: FINDING THE NEXT TERM**

Find the next four terms of the arithmetic sequence $-8, -6, -4, ...$

**Practice**

Find the next four terms of each arithmetic sequence.

1) 106, 111, 116, ...

2) $-28, -31, -34, ...$

Find the first five terms of each arithmetic sequence described.

3) $a_1 = 101, d = 9$

4) $a_1 = 210, d = -40$
The $n$th term of $a_n$ of an arithmetic sequence with first term $a_1$ and common difference $d$ is given by

$$a_n = a_1 + (n-1)d$$

Where $n$ is any positive integer.

**EXAMPLE 2: FINDING A PARTICULAR TERM**

The table shows typical costs for a construction company to rent a crane for one, two, three, or four months. Assuming that the arithmetic sequence continues, how much would it cost to rent the crane for 24 months?

<table>
<thead>
<tr>
<th>Months</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$75,000</td>
</tr>
<tr>
<td>2</td>
<td>$90,000</td>
</tr>
<tr>
<td>3</td>
<td>$105,000</td>
</tr>
<tr>
<td>4</td>
<td>$120,000</td>
</tr>
</tbody>
</table>

**PRACTICE**

Find the indicated term of each arithmetic sequence.

5) $a_1 = 4, d = 6, n = 14$  
6) $a_1 = 80, d = -8, n = 21$  
8) $a_n$ for $0, -3, -6, -9, ...$

**EXAMPLE 3: WRITE AN EQUATION FOR THE NTH TERM**

a) Write an equation for the nth term of the arithmetic sequence $-8, -6, -4, ...$
b) 256 is the _________ term of this sequence.

PRACTICE

Write an equation for the nth term of each arithmetic sequence.

9) 18, 25, 32, 39, ...

10) 6.2, 8.1, 10.0, 11.9, ...
Homework #44: Arithmetic Sequences

Determine if the sequence is arithmetic. If it is, find the common difference.

1) 35, 32, 29, 26, ...
2) −3, −23, −43, −63, ...
3) −34, −64, −94, −124, ...
4) −30, −40, −50, −60, ...
5) −7, −9, −11, −13, ...
6) 9, 14, 19, 24, ...

Given the explicit formula for an arithmetic sequence find the first five terms and the term named in the problem.

7) \(a_n = -11 + 7n\)  
   Find \(a_{34}\)
8) \(a_n = 65 - 100n\)  
   Find \(a_{39}\)
9) \(a_n = -7.1 - 2.1n\)  
   Find \(a_{27}\)
10) \(a_n = \frac{11}{8} + \frac{1}{2}n\)  
    Find \(a_{23}\)

Given the first term and the common difference of an arithmetic sequence find the first five terms and the explicit formula.

11) \(a_1 = 28, \ d = 10\)
12) \(a_1 = -38, \ d = -100\)
13) \(a_1 = -34, \ d = -10\)
14) \(a_1 = 35, \ d = 4\)
Lesson #45: Geometric Sequences

A geometric sequence is a sequence in which each term after the first is found by multiplying the previous term by a constant $r$, called the common ratio.

EXAMPLE 1: FINDING THE NEXT TERM

Find the next three terms of the geometric sequence $324, 108, 36, 12,...$.

PRACTICE

Find the next two terms of each geometric sequence.

1) $6, 12, 24,...$  
2) $2000, -1000, 500,...$

Find the first five terms of each geometric sequence described.

3) $a_1 = \frac{1}{9}, r = 3$  
4) $a_1 = 240, r = -\frac{3}{4}$
\( n^{th} \) Term of a Geometric Sequence

The \( n^{th} \) term of \( a_n \) of a geometric sequence with first term \( a_1 \) and common ratio \( r \) is given by

\[
a_n = a_1 \cdot r^{n-1}
\]

Where \( n \) is any positive integer.

**EXAMPLE 2: FINDING A PARTICULAR TERM**

Find the sixth term of a geometric sequence with which \( a_1 = -3 \) and \( r = -2 \).

**EXAMPLE 3: FINDING A TERM GIVING ANOTHER TERM AND THE RATIO**

Find the seventh term of a geometric sequence for which \( a_3 = 96 \) and \( r = 2 \).

**PRACTICE**

Find the indicated term of each geometric sequence.

5) \( a_1 = -10, r = 4, n = 2 \) \hspace{1cm} 6) \( a_1 = -6, r = -\frac{1}{2}, n = 8 \)

7) \( a_4 = 16, r = 2, n = 10 \) \hspace{1cm} 8) \( a_4 = -54, r = -3, n = 6 \)
EXAMPLE 4: WRITE AN EQUATION FOR THE NTH TERM

Write an equation for the nth term of the geometric sequence 5, 10, 20, 40, ...

PRACTICE

Write an equation for the nth term of each geometric sequence.

9) 500, 350, 425, ...
10) 8, 32, 128, ...
11) 11, –24.2, 53.24, ...
Homework #45: Geometric Sequences

Find the next two terms of each geometric sequence.
1) 405, 135, 45, ...
2) 1.4, −3.5, 8.75, ...

Find the first five terms of each geometric sequence described.
3) \(a_1 = 2, \ r = -3\)
4) \(a_1 = 243, \ r = \frac{1}{3}\)
5) \(a_1 = 576, \ r = -\frac{1}{2}\)

6) If \(a_n = 12 \left(\frac{1}{2}\right)^{n-1}\), what is \(a_7\)?

Find the indicated term of each geometric sequence.
7) \(a_1 = \frac{1}{3}, \ r = 3, \ n = 8\)
8) \(a_1 = 16,807, \ r = \frac{3}{7}, \ n = 6\)

9) \(a_8\) for 4, −12, 36, ...
10) \(a_6\) for 540, 90, 15, ...

Write an equation for the \(n\)th term of each geometric sequence.
11) 36, 12, 4, ...
12) −2, 10, −50, ...

13) What is the difference between an arithmetic sequence and a geometric sequence?